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Thermo-mechanical models of steel solidification based on two

elastic visco-plastic constitutive laws

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ABSTRACT

Two thermo-mechanical models based on different elastic-visco-plastic constitutive laws are applied to simulate temperature and stress development of a slice through the solidifying shell of 0.27%C steel in a continuous casting mold under typical commercial operating conditions with realistic temperature dependant properties. A general form of the transient heat equation, including latent-heat from phase transformations such as solidification and other temperaturedependent properties, is solved numerically for the temperature field history. The resulting thermal stresses are solved by integrating the elastic-visco-plastic constitutive laws of Kozlowski [1] for austenite in combination with the Zhu power-law [2] for delta-ferrite with ABAQUS [3] using a user-defined subroutine UMAT [4], and the Anand law for steel [5,6] using the integration scheme recently implemented in ANSYS [7]. The results from these two approaches are compared and CPU times are benchmarked. A comparison of one-dimensional constitutive

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behavior of these laws with experimental tensile test data [8,9] and previous work [10] shows reasonable agreement for both models, although the Kozlowski – Zhu approach is much more accurate for low carbon steels. The thermo-mechanical models studied here are useful for efficient and accurate analysis of steel solidification processes using convenient commercial software.

KEY WORDS: Thermomechanical processes, solidification and melting, elastic-viscoplastic material, finite elements, constitutive models, steel casting

1. Introduction

Many manufacturing and fabrication processes such as foundry shape casting, continuous casting and welding have common solidification phenomena. Probably one of the most important and complex among these is continuous casting, which is used to produce over 90% of the steel in the world today. Although the quality of continuous-cast steel is constantly improving, there is always incentive to lower the amount of defects and to improve productivity. Many of the more important defects that plague the continuous casting process are cracking problems. Many of these cracking problems are related to mismatch between solidification shrinkage and mold taper, that causes interfacial gaps and reduced heat flow between the shell and mold, leading to locally hot and thin parts of shell. These often cause transverse stresses, leading to longitudinal cracks at the meniscus, and breakouts due to ferrostatic pressure from the liquid phase applied to the newly solidified shell at mold exit [11,12]

Many of these phenomena occur during the early stages of solidification in the mold. Accurate determination of temperature, deformation and stress distributions during this time is important for correct prediction of the taper to avoid these cracking problems, in addition to understanding

other cracks, surface defects, and quality problems in the continuous casting of steel and other processes.

The high cost of plant experiments under the harsh operating steel plant conditions makes it appropriate to use all available methods in simulating, optimizing, and designing this process. Although continuous casting has been subject to many computational models, the complexity of the phenomena, including temperature, strain-rate, and phase-transformation-dependent constituitive behavior, make it difficult to model accurately. Improvements to the process to avoid cracks, such as optimizing mold taper designs, demand quantitative models that can make accurate predictions of thermal stress and strain during solidification.

The constitutive models used in previous work to investigate thermal stresses during continuous casting first adopted simple elastic-plastic laws [13,14,15]. Later, separate creep laws were added [16,17]. With the rapid advance of computer hardware, more computationally challenging elastic-viscoplastic models have been used [2,4,18,19,20,21] which treat the phenomena of creep and plasticity together since only the combined effect is measurable. Most previous models adopt a Langrangian description of this process with a fixed mesh, although an alternative mechanical model based on Eularian-Langrangian description has been proposed recently [10,21]. Similarly, the integration of viscoplastic laws ranges from easy-to-implement explicit methods [22,23], to robust but complex implicitly based algorithms [2,4,10,20].

It is a considerable challenge to apply these previous in-house FE models to solve realistic problems, which demand the incorporation of other important phenomena such as contact, thermal-mechanical coupling, and three-dimensional complexities. On the other hand, the easy-to-use commercial finite-element packages are now fully capable of handling these related phenomena, having rich element libraries, fully imbedded pre- and post- processing capabilities, advanced modeling features such as contact algorithms, and can take full advantage of parallel-computing capabilities.

The work of Koric et al [4,24,25] implemented a robust local viscoplastic integration schemes from an in-house code CON2D [2,11,12,20] into the commercial finite element package ABAQUS via its user defined material subroutine UMAT including the special treatment of liquid/mushy zone. This opened the door for the realistic computational modeling of complex steel solidification processes with ABAQUS [24,25] based on the Kozlowski III viscoplastic law for austenite, and the Zhu enhanced power law for delta ferrite phase [2]. The thermal-mechanical predictions of this model were based on measured tensile-test and creep data and have been rigorously validated against analytical solutions, a reliable in-house code [4], and with plant measurements [24].

Another finite-element commercial package ANSYS has recently implemented a different viscoplastic material, originally proposed by Anand [5] and Brown et al. [6] for the hot working of metals. Huespe et al [10] compared these two visco-plastic constitutive models of steel and concluded that the Kozlowski model was slightly more accurate and convenient than the Anand model. However, that study considered only one steel carbon content, used an in-house code with limited features and availability, and did not compare execution times.

The object of this article is to compare temperature and stress results from the Anand material model in ANSYS against those of the Kozlowski / Zhu material model using ABAQUS. In this work, a real world simulation of a typical continuous casting process is performed with both codes using realistic temperature-dependant properties on a simple slice domain. To enable a fair comparison of the crucial thermo-mechanical results developing during steel solidification using the different constitutive models, other important phenomena such as complex mold geometries, contact between the mold and strand with gap dependant conductivity, ferrostatic pressure, mold taper etc. are avoided in this paper, although they are being modeled with both of these general purpose codes in related work.

2. Thermal Governing Equations

Using an uncoupled approach, the heat conduction equation is solved first in a fixed-mesh domain that initially contains only liquid. The resulting temperature solution is then input to the subsequent mechanical analysis. The local form of the transient energy equation is given in equation (1), [23].

$$\rho\left(\frac{\partial \mathbf{H}(\mathbf{T})}{\partial t}\right) = \boldsymbol{\nabla} \cdot \left(\mathbf{k}(\mathbf{T})\boldsymbol{\nabla}\mathbf{T}\right) \tag{1}$$

along with boundary conditions:

Prescribed temperature on
$$A_T$$

Prescribed surface flux on A_q
Surface convection on A_h
 $T = T(\mathbf{x}, t)$
 $(-k\nabla T) \cdot \mathbf{n} = \hat{q}(\mathbf{x}, t)$ (1a)

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Where ρ is density, k is isotropic temperature dependant conductivity, H is temperature dependant enthalpy, which includes the latent heat of solidification. \hat{T} is a fixed temperature at the boundary $A_{T,}$ \hat{q} is prescribed heat flux at the boundary A_{q} , h is film convection coefficient prescribed at the boundary A_{h} where T_{∞} is the ambient temperature, and **n** is the unit normal vector of the surface of the domain.

3. Mechanical Governing Equations

The strains which dominate thermo-mechanical behavior during solidification are on the order of only a few percent, or cracks will form [26]. Thus, the assumption of small strain is adopted in this work. Several previous solidification models [2,17,20,21] confirm that the solidified metal undergoes only small deformation during initial solidification in the mold. With displacement spatial gradient, $\nabla \mathbf{u} = \partial \mathbf{u} / \partial \mathbf{x}$ being small, $\nabla \mathbf{u} : \nabla \mathbf{u} \approx 1$ and the linearized strain tensor is thus [27]:

$$\boldsymbol{\varepsilon} = \frac{1}{2} [\boldsymbol{\nabla} \mathbf{u} + (\boldsymbol{\nabla} \mathbf{u})^{\mathrm{T}}]$$
⁽²⁾

where Cauchy stress tensor is identified with the nominal stress tensor σ , and **b** is the body force density with respect to initial configuration.

$$\nabla \cdot \boldsymbol{\sigma}(\mathbf{x}) + \mathbf{b} = 0 \tag{3}$$

The boundary conditions are:

$$\mathbf{u} = \hat{\mathbf{u}} \qquad \text{on } \mathbf{A}_{u}$$

$$\mathbf{\sigma} \cdot \mathbf{n} = \mathbf{\Phi} \quad \text{on } \mathbf{A}_{\Phi} \qquad (3a)$$

where prescribed displacements $\hat{\mathbf{u}}$ on boundary surface portion A_u , and boundary surface tractions $\boldsymbol{\Phi}$ on portion A_{Φ} define a quasi-static boundary value problem. The rate representation of total strain in this elastic-viscoplastic model is given by:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}_{el} + \dot{\boldsymbol{\varepsilon}}_{ie} + \dot{\boldsymbol{\varepsilon}}_{th}$$
(4)

where $\dot{\boldsymbol{\varepsilon}}_{el}, \dot{\boldsymbol{\varepsilon}}_{ie}, \dot{\boldsymbol{\varepsilon}}_{th}$ are the elastic, inelastic (plastic + creep), and thermal strain rate tensors respectively. Stress rate $\dot{\boldsymbol{\sigma}}$ depends on elastic strain rate, and for a linear isotropic material with negligible large rotations, is given by equation (5) in which ":" represents inner tensor product.

$$\dot{\boldsymbol{\sigma}} = \underline{\mathbf{D}} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_{ie} - \dot{\boldsymbol{\varepsilon}}_{th}) \tag{5}$$

 $\underline{\mathbf{D}}$ is the fourth order isotropic elasticity tensor given by equation (6).

$$\underline{\underline{\mathbf{D}}} = 2\mu \underline{\underline{\mathbf{I}}} + (k_{\rm B} - \frac{2}{3}\mu)\mathbf{I} \otimes \mathbf{I}$$
(6)

Here μ , k_B are the shear modulus and bulk modulus respectively and are in general functions of temperature, while $\mathbf{I}_{=}, \mathbf{I}$ are fourth and second order identity tensors and " \otimes " denotes outer tensor product.

3.1 Viscoplastic Strain Models

Viscoplastic strain includes both strain-rate independent plasticity and time dependant creep. Creep is significant at the high temperatures of the solidification processes and is indistinguishable from plastic strain [20]. Kozlowski et al [1] proposed a unified formulation with the following functional form to relate inelastic strain to stress, temperature, strain rate, and carbon content in the austenite phase of steel.

$$\dot{\overline{\epsilon}}_{ie} = f(\overline{\sigma}, T, \overline{\epsilon}_{ie}, \%C)$$
(7)

the equivalent inelastic strain-rate $\dot{\overline{\epsilon}}_{ie}$ is a function of equivalent stress $\overline{\sigma}$, temperature T, equivalent inelastic strain $\overline{\epsilon}_{ie}$, and steel grade defined by its carbon content %C.

$$\overline{\sigma} = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$$
(8)

 σ ' is a deviatoric stress tensor defined in equation (9).

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}$$
⁽⁹⁾

The particular model below was chosen to match tensile test measurements of Wray [9] and creep test data of Suzuki et al [28] for plain carbon steel in the austenite phase.

$$\dot{\overline{\epsilon}}_{ie} = f_{C} \left(\overline{\sigma} - f_{1} \overline{\epsilon}_{ie} \mid \overline{\epsilon}_{ie} \mid \overline{\epsilon}_{i} \mid f^{2-1} \right)^{f^{3}} exp\left(-\frac{Q}{T}\right)$$

where :

$$Q = 44,465$$

$$f_{1} = 130.5 - 5.128 \times 10^{-3} T$$

$$f_{2} = -0.6289 + 1.114 \times 10^{-3} T$$

$$f_{3} = 8.132 - 1.54 \times 10^{-3} T$$

$$f_{C} = 46,550 + 71,400 (\%C) + 12,000 (\%C)^{2}$$
(10)

Q is activation constant, and f_1, f_2, f_3, f_c are empirical functions of temperature or steel-grade, equivalent stress $\overline{\sigma}$ is given in MPa, and temperature T in K.

To simulate the delta-ferrite phase of steel, a power-law constitutive model, was proposed by Zhu [2] which generates the much higher creep rates experienced in this body-centered cubic phase, relative to the strong, face-centered cubic austenite phase. This constitutive model, given in equation (10a) was based on tensile test measurements by Wray [8]. It is applied in the solid whenever the volume fraction of ferrite is more than 10%. Otherwise, equation (10) is applied. This simple rule was preferred over a mixture rule based on phase fraction, because creep in the delta-ferrite phase dominates the mechanical behavior if this phase is continuous. The volume fractions of each phase are calculated from an iron-carbon phase diagram adjusted for other alloying components (1.52%Mn, 0.34%Si, 0.015%S, and 0.012%P), as implemented in the inhouse code, CON2D [20].

$$\dot{\overline{\varepsilon}}_{ie} = 0.1 \left| \frac{\overline{\sigma}}{f_{\delta e} \left(\frac{T}{300} \right)^{-5.52} (1 + 1000 \overline{\varepsilon}_{ie})^{m}} \right|^{n}$$

$$w h ere :$$

$$f_{\delta e} = 1.3678 \times 10^{4} (\% C)^{-5.56 \times 10^{-2}}$$

$$m = -9.4156 \times 10^{-5} T + 0.3495$$

$$n = \frac{1}{1.617 \times 10^{-4} T - 0.06166}$$
(10a)

Again equivalent stress $\overline{\sigma}$ is given in MPa, and temperature T in K in equation (10a).

A different viscoplastic model for steel at high temperature was proposed by Anand [5] and Brown at el. [6]. Like the Kozlowski model, there is no explicit yield surface, as the instantaneous material response depends only on its current state. A single scalar variable s, called the deformation resistance, is used to represent the isotropic resistance to inelastic strain. The constitutive equation is given in equation (11).

$$\dot{\overline{\epsilon}}_{ie} = A_A \exp\left(-\frac{Q_A}{T}\right) \left[\sinh\left(\xi\frac{\overline{\sigma}}{s}\right)\right]^{\frac{1}{m}}$$
(11)

The evolution equations for s are

$$\dot{s} = h_0 \left(\left| 1 - \frac{s}{s^*} \right|^a \operatorname{sign}(1 - \frac{s}{s^*}) \right) \dot{\overline{\epsilon}}_{ie}$$
(12)

with

$$s^* = \tilde{s} \left[\frac{\overline{\dot{c}}_{ie}}{A_A} \exp\left(\frac{Q_A}{T}\right) \right]^n$$
(13)

where:

S	[Pa]	deformation resistance
Q _A	[K]	activation energy over gas constant for Anand's material
A _A	[1/sec]	pre-exponential factor
ξ		multiplier of stress
m		strain rate sensitivity of stress
ho	[Pa]	hardening/softening constant
ŝ	[Pa]	saturation value for s
n		strain rate sensitivity of saturation
a		strain rate sensitivity of hardening or softening

In addition, an initial value for deformation resistance s_{o} must be defined.

Using the experimental data of Wray [9], Anand [5] estimated the parameters for carbon steel in a carbon content range 0.05-0.5 %C. The current Anand model implemented in ANSYS has been slightly modified from the original with the addition of a hyperbolic sine functional form of the constitutive equation and exponential hardening behavior. The standard material constants used for this model in this work are listed in Table 1. Brown et al. [6] proposed the initial value for deformation resistance s_0 to depend on temperature, while the initial work of Anand [5] defined s_0 to vary in the range of 35-52 MPa, depending on both temperature and strain rate. No temperature or composition dependence of any of these model parameters is currently available in ANSYS, so the average value of 43 MPa is chosen for s_0 following the work of Huespe at al. [10].

The Kozlowski model, on the other hand, has no adjustable parameters. For lower-carbon steels involving delta-ferrite, however, the Kozlowski model for austenite should be combined with a separate power law equation (10a) for temperatures at which delta-ferrite is present. Details of the complete phase-dependent constitutive equations are given elsewhere [2,30].

The steels considered in this work are assumed to harden isotropically, so the von Mises loading surface, associated plasticity, and normality hypothesis of the Prandtl-Reuss flow law, equation (14), [29] is used to calculate visco-plastic strain components .

$$(\dot{\varepsilon}_{ie})_{ij} = \frac{3}{2} \, \overline{\dot{\varepsilon}}_{ie} \, \frac{\sigma'_{ij}}{\overline{\sigma}} \tag{14}$$

3.2 Thermal Strain

Thermal strains ε_{th} arise due to volume changes caused by both temperature differences and phase transformations, including solidification and solid-state phase changes between crystal structures, such as austenite and ferrite.

$$\left(\varepsilon_{\rm th}\right)_{\rm ij} = \int_{T_0}^{T} \alpha(T) dT \,\delta_{\rm ij} \tag{15}$$

where α is temperature dependant coefficient of thermal expansion, T_0 is an important reference temperature and δ_{ij} is Kronecker's delta. The choice of T_0 is arbitrary, but it significantly affects the associated α function.

4. Local Time Integration of the Inelastic Constitutive Models

Owing to the highly strain-dependant inelastic responses, a robust integration scheme is required to integrate either the Kozlowski or Anand equations over a generic time increment Δt . The system of ordinary differential equations defined at each material point by the Kozlowski model equation (10) or the Zhu power law model equation (10a) is converted into two "integrated" scalar equations by the Euler backward method and then solved using a special bounded Newton-Raphson method [2,4,30]. Details of this local integration scheme can be found at [2,4,30] along with the derivation of the Jacobian consistent with this method.

Similarly, ANSYS uses the Euler-backward scheme to integrate equations (11) and (12), [7]. The details of this local integration scheme that is built into ANSYS and specially optimized for the Anand model are not publicly available.

The solution obtained from this "local" integration step from all material (gauss) points is used to update the global finite-element equilibrium equations, which are solved using the Newton-Raphson based nonlinear finite-element procedures in ABAQUS or ANSYS [3,7].

5. Comparison of constitutive models with experimental data

The two constitutive models were first evaluated for spatially-uniform conditions, by simply integrating the equations with a local method. Fig. 1 compares the calculated tensile curves with experimental data of Wray [9] for different carbon contents. The Kozlowski model correctly captures the slight softening effect of increasing carbon content for this fully-austenitic condition. Lacking any dependency on steel grade, the Anand model is represented with a single curve, which underestimates stress for the low and mild carbon content steels, and underestimates work hardening, as indicated by the flatness of the curves.

Fig. 2 compares the stresses at 5 pct strain measured by Wray [8] at different temperatures to those predicted with the Kozlowski austenite model or Zhu power law for delta ferrite and the Anand model. Both model systems exhibit the correct drop in stress when integrated at lower constant strain rate. The experiments and Kozlowski / Zhu model predictions in this figure both show that delta-ferrite, which forms in low carbon steels at higher temperatures, is much weaker than austenite. This important effect of phase explains the lower stress measured in feritic Sisteel at lower temperature, while the ultra-low carbon steel and Si-steel show similar stresses in the fully-feritic region above 1400 °C. The Anand model fails to capture this significant change in mechanical behavior of low carbon steel shells containing delta-ferrite.

For the 1030 steel, Huespe et al. [10] showed that the Kozlowski model has a generally better fit with available experimental data of Suzuki [28], while the Anand model showed a slightly better agreement with experimental data of Wray [9]. However, due to the uncertainty of s_0 and lack of dependency on carbon content %C in the Anand model, it was concluded in that work that the Kozlowski model is better. A recent survey of various constitutive models of steel at elevated temperature conducted by Pierer et al [31] has found that the Kozlowski model produces the closest match with experimental steel solidification force-elongation curves. Additional

information on these models, including further comparison with experimental measurements can be found in the following papers: [1,5,6,10,31,32]

6. Analysis of solidifying shell in continuous casting mold

In many solidification processes, such as the continuous casting of steel, one dimension of the casting is much longer than the others, and is otherwise unconstrained. In this case, it is quite reasonable to apply a condition of generalized plane strain in the long (axial) direction (z), and to solve a two-dimensional thermal stress problem in the transverse (x-y) plane. This condition reasonably allows a two-dimensional transient mechanical computation in the plane section to produce the complete three-dimensional stress state in the casting. While generalized-plane-strain elements are available in ABAQUS, the current implementation of so called visco elements, which only works with Anand's material in ANSYS, does not support generalized plane strain. Therefore, this comparative investigation employs a simple plane-strain implementation in both codes.

The domain adopted for this problem is a thin slice through the shell thickness given in Figure 3. For the heat conduction computation, the high Peclet number ($V_c L \rho c_p / k$) associated with the high casting speed (V_c) and low thermal conductivity (k) of steel continuous casting makes axial conduction negligible relative to axial advection [33,20]. Thus, the same simple slice domain that moves with the strand in a Langrangian frame of reference can be used for both the heat transfer and mechanical computations. Fig. 4 shows the domain and boundary conditions for both models. An instantaneous interfacial heat flux profile that varies with time down the mold according to mold thermocouple measurements [20] is given in Fig. 5, and is applied at the left edge of the domain. Due to the large width (x) of the casting compared to the thickness (y) of this simple domain, a second generalized plane strain state is applied in the y direction. This condition was imposed by coupling the displacements of all nodes along the bottom edge of the slice domain. This was accomplished using the *EQUATION option in ABAQUS [3], and the CP command in ANSYS [7]. The normal (x) displacement of all nodes along the bottom edge of the domain is fixed to zero. Tangential stress was left equal zero along all surfaces. Finally, the ends of the domain are constrained to remain vertical, which prevents any bending in the xy plane.

Temperature-dependent properties were chosen for %0.27C plain mild-carbon steel with T_{sol} =1411.79 °C and T_{liq} =1500.72 °C (solidus and liquids temperatures). The enthalpy curve used to relate heat content and temperature in this study, H(T), is shown in Fig. 6. It was obtained by integrating the specific heat curve fitted from measured data of Pehlke et. al. [34]. While this enthalpy data is input directly into ANSYS, ABAQUS tracks the latent heat Hf=257,867 J/kg separately from the specific heat c_p , which is found from the slope of this H curve, except in the solidification region, where c_p is found from [23] as follows

$$c_{p} = \frac{dH}{dT} - \frac{Hf}{\left(T_{liq} - T_{sol}\right)}$$
(16)

The temperature dependent conductivity function for 0.27%C plain carbon steel is fitted from data measured by Harste [35], and is given in Fig. 7. The conductivity increases in the liquid region by a factor of 6.65 to partly account for the effect of convection due to flow in the liquid steel pool [36]. Density was assumed constant at this work, 7400 kg/m³, in order to maintain constant mass.

The temperature-dependant coefficient of thermal expansion $\alpha(T)$ is calculated from the thermal linear expansion function TLE [20] with a reference temperature of To=1540 °C, and is given in Fig. 8. An alternative, exactly-equivalent thermal-expansion function is included in this figure using a reference temperature of To=Tsol=1411.79 °C.

Poisson ratio is 0.3 constant. Elastic modulus E generally decreases as the temperature increases, although its value at very high temperatures is uncertain. The temperature-dependent elastic modulus curve used in this model was fitted from measurements from Mizukami et. al. [37], as shown in Fig. 9. The liquid and mushy zone is modeled by lowering elastic modulus by three orders of magnitude. This method is easy to apply but can not model the generation of inelastic strain and stress in the liquid/mushy zone which is crucial for hot tearing prediction [20]. It also sometimes introduces a numerical ill-conditioning of the global stiffness matrix after finite-element assembly which might be a problem for sparse linear solvers. Other more sophisticated liquid/mushy models have been proposed by Zhu [2], Li [20], and Koric [4].

A 20 sec. simulation was performed, which corresponds to a 670 mm long shell of steel cast at a casting speed of 2 m/min. The heat transfer analysis is run first to get the temporal and spatial temperature field. Stress analysis is then run using this temperature field. The domain used in both codes has a single row of 300 plane 4-node elements in both thermal and stress analysis. A formal study of mesh and time increment refinement was conducted by Zhu [2], which shows that the 300-node mesh used here is more than sufficient to achieve accuracy within 1% error with a fixed time increment of 0.01 sec (1000 time increments per 10 s) compared to the analytical solidification solution for the elastic-perfectly plastic material [38].

7. Results and Discussion

The temperature results predicted with ABAQUS and ANSYS are in excellent agreement, as shown in Figs. 10 and 11. Considering that the two codes employ different forms of thermal parameters (H and k in ANSYS; and T_{sol} , T_{liq} , H_f, and k in ABAQUS), this shows that both sets of thermal material properties are consistent. Furthermore, the two numerical implementations are equivalent.

The temperature gradient through the shell is almost linear from near the solidification front to the cooled surface and it gradually drops as solidification proceeds. The typical cooling histories for two material points in Fig. 11 each show the classic drop in cooling rate as each point beneath the surfaces passes through the "mushy region" between the solidus and liquidus temperatures. The solidification front grows roughly parabolically with time, which matches both theoretical expectations and plant measurements [33].

The total lateral (y) shrinkage strain history given in Fig. 12 for the bottom edge nodes also shows a very good agreement between 2 models. This shrinkage displacement is the same across the entire domain, and shows the decrease in the average width of the solidifying shell, which is accommodated in practice by tapering the mold walls. This result represents a prediction of ideal taper, and shows that more taper is needed near the beginning of solidification in the top region of the mold. This calculation is relatively insensitive to the constitutive model, because the shrinkage is predominantly thermal strain, and can be reasonably approximated by simple thermal strain calculations [39].

The stress predictions, given in Figs. 13 - 14 match reasonably well at early times, but start to diverge with increasing time. For both models, the faster cooling of the interior relative to the surface region naturally causes interior contraction and tensile stress, which is offset by compression at the surface. The Anand model underpredicts both the compressive surface stress and the internal tensile peak. This finding is consistent with the stress underprediction from Fig. 1 as well as with the axial stress results from the in-house code of Huespe at al. [10] for round billet casting under different conditions. These results indicate the earlier observed differences between the two constitutive models, which increase with decreasing temperature. Qualitatively, however, both models reasonably predict thermal-mechanical behavior during solidification, and can provide insights into casting processes.

Finally, the wall clock times of the two codes in this work are comparable. The Anand model with ANSYS was faster, taking 3.5 min, versus 5.5 min. for the Kozlowski / Zhu model with

ABAQUS. Both simulations were performed on the IBM p690 platform with a Power 4, 1.3 GHz CPU.

8. Conclusions

Temperature and stress in a solidifying slice through a realistic steel continuous caster are predicted with two different elastic-visco-plastic constitutive laws for plain-carbon steel using two commercial finite-element programs. The Anand law is integrated by the Euler-Backward method built into ANSYS. The results are compared with the Kozlowski model for austenite combined with the Zhu power-law model for delta-ferrite, integrated in ABAQUS with a local-global integration scheme implemented via a user-defined UMAT subroutine.

While the temperature and total strain results are in excellent agreement, the Anand model underpredicts the peak stresses in both compression and tension. The results are consistent with the tensile stress comparisons in Fig. 1 as well as the findings of previous work [10] using an inhouse code. The Anand model with ANSYS qualitatively predicts the expected thermalmechanical behavior with the least CPU time. However, the Kozlowski / Zhu model has been validated with experimental measurements, accurately incorporates steel grade dependency, needs no adjustable parameters to be defined, and can utilize the generalized plane-strain condition in ABAQUS. In addition, only the Kozlowski / Zhu model correctly predicts the weakening behavior of delta ferrite, which forms near the solidification front in low carbon steels. In conclusion, both ANSYS and ABAQUS enable modeling of complex realistic casting phenomena including variable interfacial gap heat transfer, ferrostatic pressure from the liquid, thermo-mechanical contact between the mold and strand, mold taper, and complex threedimensional geometric features. Two efficient and convenient approaches are available to investigate thermal-mechanical behavior involving the solidification of steel, especially while in the austenite phase.

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Figures and Tables



Fig. 1. Tensile stress curves calculated with Kozlowski and Anand models for various carbon content and compared to Wray experimental data



Fig. 2. Constitutive model comparison with Wray experimental data for low carbon steel, showing mechanically weaker delta ferrite phase



Fig. 3. Solidifying slice



Heat Transfer Slice Domain and BC-s

Fig. 4. Mechanical and thermal finite element domains



Fig. 6. Enthalpy for 0.27 %C plain carbon steel



Fig. 8. Coefficient of thermal linear expansion for 0.27%C plain carbon steel, reference temperatures: To=1540 °C and To=1411.7 °C



Fig. 10. Temperature distribution along the solidifying slice in continuous casting mold



Fig. 11. Temperature history for the surface material point and the material point 5 mm from the surface





Fig. 13. Lateral (y) stress distribution along the solidifying slice in continuous casting mold



Fig. 14. Lateral (y) stress history for the surface material point and the material point 5 mm from the surface

Parameter	Value
So	43 MPa
Q _A	32514 K
A	1.E11
بخ	1.15
m	0.147
h _o	1329 MPa
ŝ	147.6 MPa
n	0.06869
a	1

Table1 Parameters used in the Anand material model for 1030 steel

Parameter	Value
Steel Casting Speed	2.2 m/min
Working Mold Length	670 mm
Carbon Content	0.27 %C
Initial Temparature	1540 °C
Liquidus Temperature	1500.70 °C
Solidus Temperature	1411.79 °C
Ref. Temperature for Thermal Expansion	1540 °C
Density	7400 kg/m ³
Poisson's Ratio	0.3

Table 2. Casting Conditions

Nomenclature

А	m^2	Surface	
A	1/sec	Anand Pre-Exponential Factor	
A _T	m ²	TempPrescribed Surface	
A _q	m ²	Flux-Prescribed Surface	
A_h	m^2	Convection-Prescribed Surface	
A_u	m ²	Displacement-Prescribed Surface	
A_{Φ}	m ²	Traction-Prescribed Surface	
a		Anand Strain Rate Sensitivity of Hardening or Softening	
b	Ν	Volumetric Force Vector	
c _p	J/kgK	Specific Heat	
₫	N/m ²	4 th Order Elasticity Tensor.	
Е	N/m ²	Elastic Modulus	
f	1/ses	Viscoplastic Law Function	
fc	MPa ^{-f3} s ⁻¹	Empirical Constant in Kozlowski III law	
$\boldsymbol{f}_{\delta c}$		Empirical Constant in Enhanced Power Delta law	
\mathbf{f}_1	MPa	Empirical Constant in Kozlowski III law	
\mathbf{f}_2		Empirical Constant in Kozlowski III law	
f_3		Empirical Constant in Kozlowski III	
Н	J/kgK	Enthalpy	
Hf	J/kgK	Latent Heat	
h	W/m ² K General Film Coefficient		
h _o	N/m ²	Anand hardening/softening constant	
Ī		4th Order Identity Tensor	
Ι		2 nd Order Identity Tensor	
k	W/mK	Thermal Conductivity	
k _B	N/m ²	Bulk Modulus	
L	m	Characteristic Axial Casting Length	
m,n		Empirical constants used power delta law	
m		Anand Strain Rate Sensitivity of Stress	

n		Anand Strain Rate Sensitivity of Saturation
n		Surface Unit Vector
ĝ	W/m^2	Prescribed Heat Flux
Q,QA	Κ	Activation Energy Constants
S	N/m^2	Anand Deformation Resistance
ŝ	N/m ²	Anand Saturation Value for s
So	N/m ²	Anand Initial Value for s
Т	°C,K	Temperature
Î	°C	Prescribed BC Temp.
T_{∞}	°C	Ambient Temperature
T_{init}	°C	Initial Temperature.
To	°C	Reference Temperature
T_{sol}	°C	Solidus Temp.
T_{liq}	°C	Liquidus Temp.
TLE		Thermal Linear Expansion
u,d	m	Displacement Vector
V	m ³	Volume
V_{c}	m/min	Casting Speed
X	m	Position Vector
Z	m	Distance Bellow Meniscus
α	1/°C	Coefficient of Thermal Expansion
$\boldsymbol{\delta}_{ij}$		Kronecker's Delta
3		Total Strain Tensor
έ	1/sec	Total Strain Rate Tensor
8 _{el}		Elastic Strain Tensor
έ _{el}	1/sec	Elastic Strain Rate Tensor
8 _{ie}		Inelastic Strain Tensor
έ _{ie}	1/sec	Inelastic Strain Rate Tensor
$\dot{\overline{\epsilon}}_{ie}$	1/sec	Equivalent Inelastic Strain
${oldsymbol{\epsilon}}_{ ext{th}}$		Thermal Strain Tensor

$\dot{\boldsymbol{\epsilon}}_{_{th}}$	1/sec	Thermal Strain Rate Tensor
μ	N/m ²	Shear Modulus
σ	N/m^2	Stress Tensor - small strain formulation
σ'	N/m ²	Deviatoric Stress Tensor
σ^{*}	N/m ²	Trial Stress Tensor
$\overline{\sigma}$	N/m ² ,MPa	Equivalent Stress
ρ	kg/m ³	Density
μ	N/m^2	Shear Modulus
ξ		Anand Multiplier of Stress
Φ	N/m^2	Surface Traction Vector
%С		Percentage Carbon in the steel